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MEASUREMENT OF DEMAGNETIZING
FACTOR OF RING-CORE

BY

758

SUBHASHCHANDRA CHIMANLAL PATEL, 1942

A
THESIS

submitted to the faculty of
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1968

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Approved by

Stanley V. Marshall (advisor) George M. Benson
Don M. Spauldin

ABSTRACT

A method has been presented to measure the demagnetizing factor of ring-cores. First the incremental apparent permeability and the incremental true permeability were measured at different degrees of saturation and then the demagnetizing factor was determined from those results.

The cores used were tape wound ring-cores of mean length of 8.0 cm. The method was found suitable for cores wound with $\frac{1}{4}$ mil or thicker tape, but was in question for the $\frac{1}{8}$ mil tape core tested.

ACKNOWLEDGEMENT

The author would like to take the privilege of expressing his indebtedness to his advisor, Dr. Stanley V. Marshall, for his guidance and encouragement and without whose help this work would not have been realized.

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I. INTRODUCTION

When a rod of ferromagnetic material is placed in an uniform magnetic field H_x , the field H within the rod is related to the ambient field by the following relation

$$H = H_x - K (B-H) \quad (1.1)$$

where K is demagnetizing factor

and B is flux density within the material.

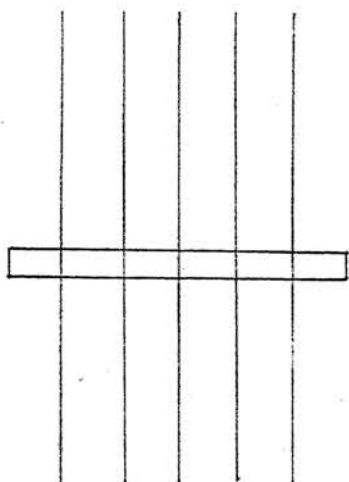
(Throughout this thesis the cgs system of units is used).

If the true permeability (μ) is very large such that terms of $1/\mu$ are negligible compared to unity¹,

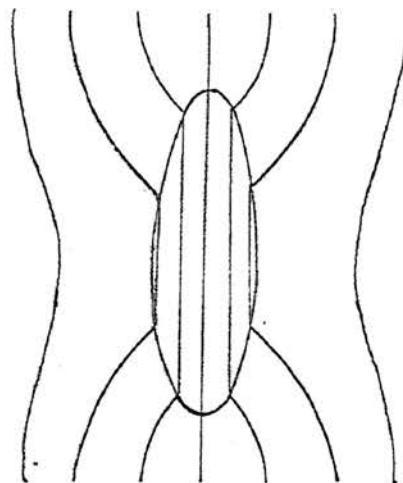
$$H_x/B = 1/\mu_a = 1/\mu + K \quad (1.2)$$

μ_a is apparent permeability of the material.

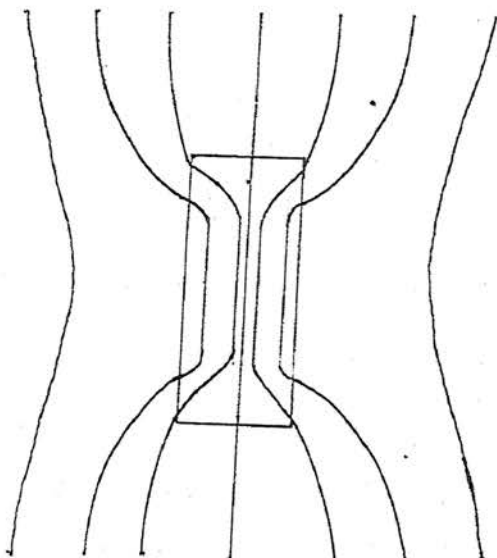
The demagnetizing factor of the ferromagnetic material is a function of the geometry of the core and not of the material itself and is smallest for the largest length to diameter ratios where the longest dimension is in the direction of the applied field. For a thin disc, the demagnetizing factor K is unity (Fig. 1). For a prolate ellipsoid, K approaches zero as the ratio of major axis to either of minor axis is increased indefinitely². For one



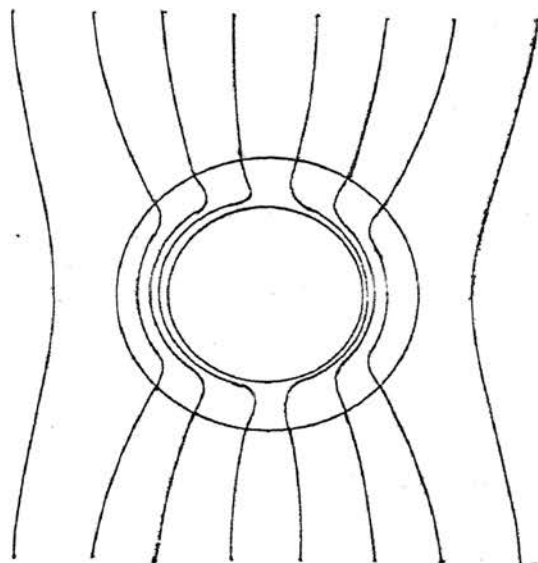
Flat Disk



Prolate Ellipsoid



Cylindrical Rod



Ring-Core

Figure 1. Flux Patterns for Magnetized Objects

special case of the general ellipsoid, that of sphere, $K = 0.333$. It is only for the general ellipsoid that the field within the material is uniform, and for other configurations, the demagnetizing factor is a function of the position within the material at which it is measured. A special kind of demagnetizing factor, a ballistic demagnetizing factor, has been applied to the cylindrical rod³ shown in Fig. 1, which has also been applied to the toroid or ring core in this thesis. The ballistic demagnetizing factor is defined by equation 1.1 if B and H are the average values over the median cross-section, and will be implied in this thesis.

An expression similar to equation 1.2 can also be derived (See appendix A),

$$1/\mu_{a\Delta} = 1/\mu_{\Delta} + K \quad (1.3)$$

where $\mu_{a\Delta}$ = Incremental apparent permeability,
and μ_{Δ} = Incremental true permeability.

Curves of K for various geometrical shapes, derived both from calculations and measurements, have been published^{4,5}. The demagnetizing factor of the toroid is one of the important factors in the design of a ring-core magnetometer. The ring-core magnetometer was first introduced in 1961 by W.A. Gayger. Several of the inherent advantages of this new approach were described by Gayger in a subsequent series of publications⁶⁻⁹. Since then, much work has been done in the field of sensitivity^{10,11}, and an

analytic model^{12,13} has been developed. The sensitivity of the closed core magnetometer is significantly influenced by the demagnetizing factor. Because of the demagnetization the input signal H_x is attenuated before detection. Consequently, the sensitivity of the magnetometer depends upon the extent of attenuation of this signal.

Demagnetizing factor is an important factor where there is an interaction between the magnetic material and an external magnetic field. The magnetometer is one example but there are many other such magnetic circuits.

This paper deals with the measurement of the demagnetizing factor of small tape-wound ring-cores using equation 1.3. Throughout this thesis the incremental permeability is assumed to be approximately equal to the differential permeability and the two terms are used interchangeably. This assumption is made for both apparent as well as true permeabilities.

The types of core and core-sizes used were as shown in Table I. All the three cores were supplied by Infinetics, Inc. They were prepared from the same material but were prepared in different gages and subjected to different heat treatments.

Table I. Core Sizes

Core Type	Inner Diameter	Nominal Tape Thickness	Number of Wraps	Width of Tape	Mean length of the core-cm.	Cross-Sectional Area - cm ²
S-1000 E47-HA-1935-C	1.00"	1/8 mil	40	0.125"	8.0	5.05×10^{-3}
S-1000 E47-HB-1936-C	1.00"	1/4 mil	20	0.125"	8.0	4.2×10^{-3}
S-1000 E47-HC-1937-C	1.00"	1/2 mil	10	0.125"	8.0	4.97×10^{-3}

II. MEASUREMENT OF APPARENT PERMEABILITY

The apparent permeability of the ring-core configuration was determined as shown in Fig. 2, using different types of ring-cores. Because of the air path portion of the signal flux path, the hysteresis of the material may be neglected and for small fields the ratio of flux density induced in each core section to the field may be considered linear. Thus the apparent permeability and apparent differential permeability, are essentially the same. For convenience, a sinusoidal voltage at 1000 Hz was used in the measurements.

The Helmholtz Coils used were square coils, four feet on a side with a separation of 2.188 feet, mounted on a tiltable horizontal axis. They were oriented in such a way as to produce a field at right angle to the earth's field. This minimized the effect of the earth's field on the saturation level of the ring-core. The coils had 500 turns each of No. 22 wire wound on a coil form of one inch aluminum channel. The inductance of the Helmholtz Coil pair, connected in series was measured to be 2.42 henries. The other data about the Helmholtz Coils are as follows:

Coil Constant - 6.65 oersteds/amp.

D.C. Resistance - 130 ohms per coil

68 to 69 MA are required to cancel the earth's field when arranged to directly oppose it.

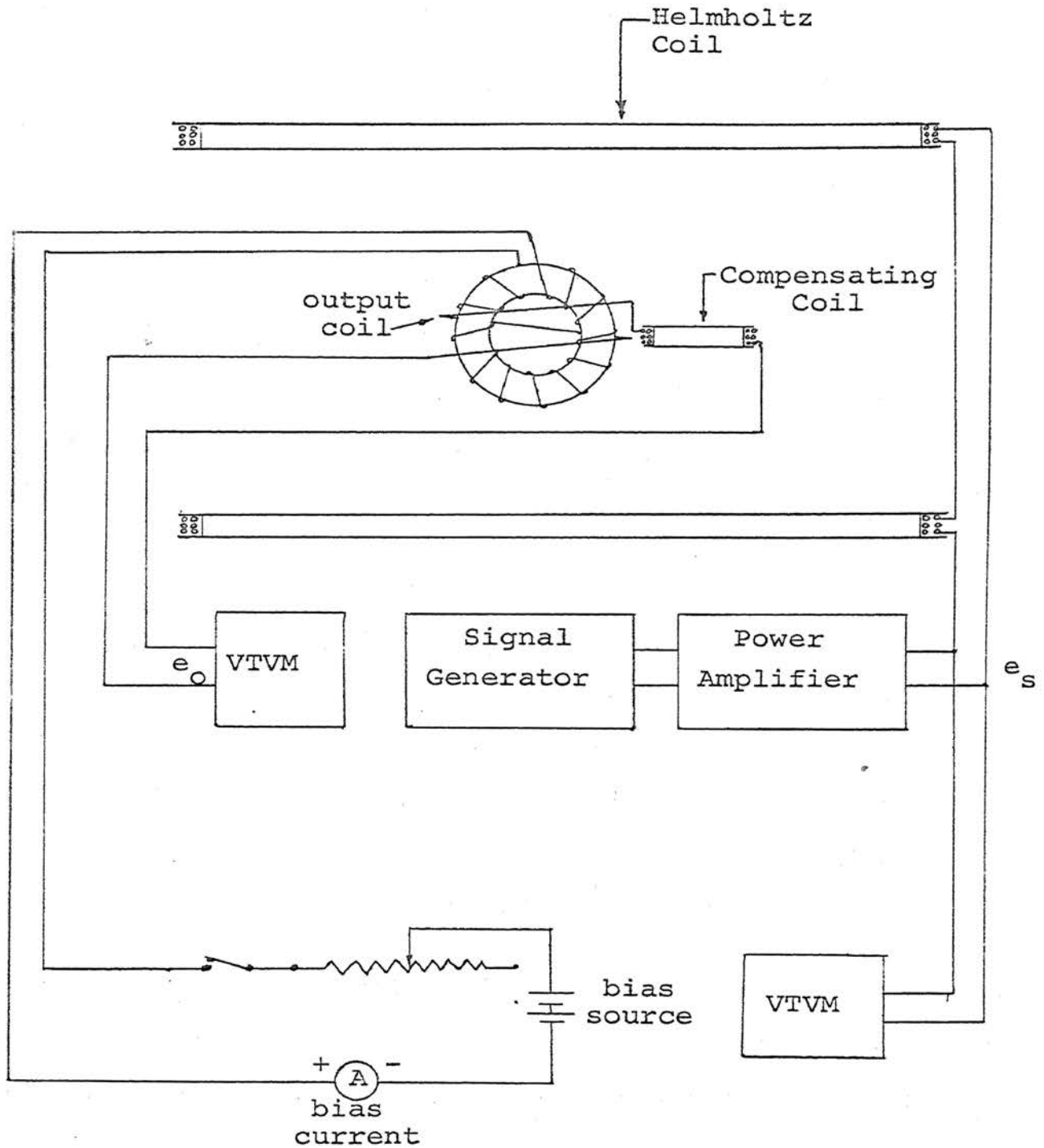


Figure 2: Experimental Circuit for Measurement of Apparent Permeability.

The output coil was wound with 850 turns on a rectangular plastic bobbin having very narrow groove. The inner dimensions of the bobbin were just large enough to accomodate the ring-core. During the experiment the plane of the output coil was kept perpendicular to the magnetic field of the Helmholtz coils and the ring-core was kept such that its center coincided with that of the output coil as shown in Fig. 2. The compensating coil was an ordinary circular coil with about 200 turns and about 100 cm² area.

Considering the circuit of Fig. 2, the following expression can be derived

(For the derivation, see Appendix B)

$$\mu_{a\Delta} = \frac{1.835 \times 10^7}{N_o A} \times \frac{E_o}{E_{is}} \quad (2.1)$$

where E_o = RMS output voltage

E_{is} = RMS input signal voltage

A = Cross sectional area of core material, cm²

N_o = Number of turns of output coil

The above equation was used to determine the incremental apparent permeability. This permeability was measured at different levels of saturation of the core. The core was saturated by passing a measured amount of direct current through a biasing winding wound uniformly around the core.

Before taking the data, the core was completely demagnetized. This was done by passing sufficiently high alternating current with gradually decreasing amplitude, through the bias winding. It was observed that the initial

demagnetization of the core was very important because of the residual magnetism.

After the initial demagnetization the compensating coil was rotated so that the output voltage was minimum with no core in the output coil. The rotation changes its effective area perpendicular to the alternating field. This adjustment minimized the error caused by the voltage induced due to the rate of change of flux in the air portion of the output coil. It was not possible to adjust the compensating coil to give zero output voltage. The minimum output voltage (E_{oi}) was used to calculate correct output voltage (E_o) from the measured output voltage (E_{om}) using the following relation

$$E_o^2 = E_{om}^2 - E_{oi}^2 \quad (2.2)$$

After proper adjustment of the compensating coil the core was inserted into the output coil and output voltage was read corresponding to different degrees of saturation keeping the signal voltage constant at 100V RMS.

The signal voltage had to be large enough to develop about 10 MV across the output coil. This gave sufficient signal-to-noise ratio to obtain accuracy in E_o . However, the signal voltage had to be small enough to justify the assumption that the a.c. field was changing over a differential range. Considering both of these points, 100 volts was judged to be a reasonable level for the signal voltage.

The variable resistance used in the bias circuit was a decade resistance box. Although this was satisfactory

it would be better to use a continuously variable resistance. This is because every time the decade resistance is varied, the current goes to zero for a very small time and then rises to some new value. This may introduce some error because of the hysteresis effect. However, if enough care is taken while using such resistances, the error may be acceptable.

A digital computer program was written to calculate $\mu_{a\Delta}$ and H from the data obtained. The important data are shown in Table II. For the computer program see Appendix C.

Table II. Apparent Permeability for Various Degrees of Excitation

Sr No	S-1000 E47-HA-1935-C		S-1000 E47-HB-1936-C		S-1000 E47-HC-1937-C	
	H Oersteds	Apparent Permeability	H Oersteds	Apparent Permeability	H Oersteds	Apparent Permeability
1	0.00	351.3	0.00	452.0	0.00	450.2
2	0.242	218.8	0.242	280.6	0.242	348.2
3	0.484	136.4	0.363	230.8	0.484	284.5
4	0.726	85.4	0.484	194.8	0.726	238.9
5	0.968	59.1	0.605	161.7	0.968	204.5
6	1.210	39.3	0.726	136.1	1.21	176.9
7	- -	- -	0.847	116.1	1.45	158.4

III. MEASUREMENT OF INCREMENTAL TRUE PERMEABILITY

As mentioned earlier, the incremental permeability was assumed to be equal to the differential permeability. The differential permeability was measured by a novel method developed by Marshall¹⁴.

Figure 3 shows the circuit used for this measurement. The supply voltage was adjusted to such a level that the core was just getting saturated at the peak voltage of the secondary side of the isolating transformer. This was assured by observing a little notch at the peak of the secondary voltage waveform.

Considering the circuit of Figure 3, during a short time near the peak of the voltage across the secondary side of the isolating transformer, a mathematical derivation (see Appendix D) gave the following relation,

$$\frac{dB}{dH} = \frac{L_1 (E - E_v - E_h \cdot R/R_1)}{C_1 (E_v - E_h R_3/R_1)} \quad (3.1)$$

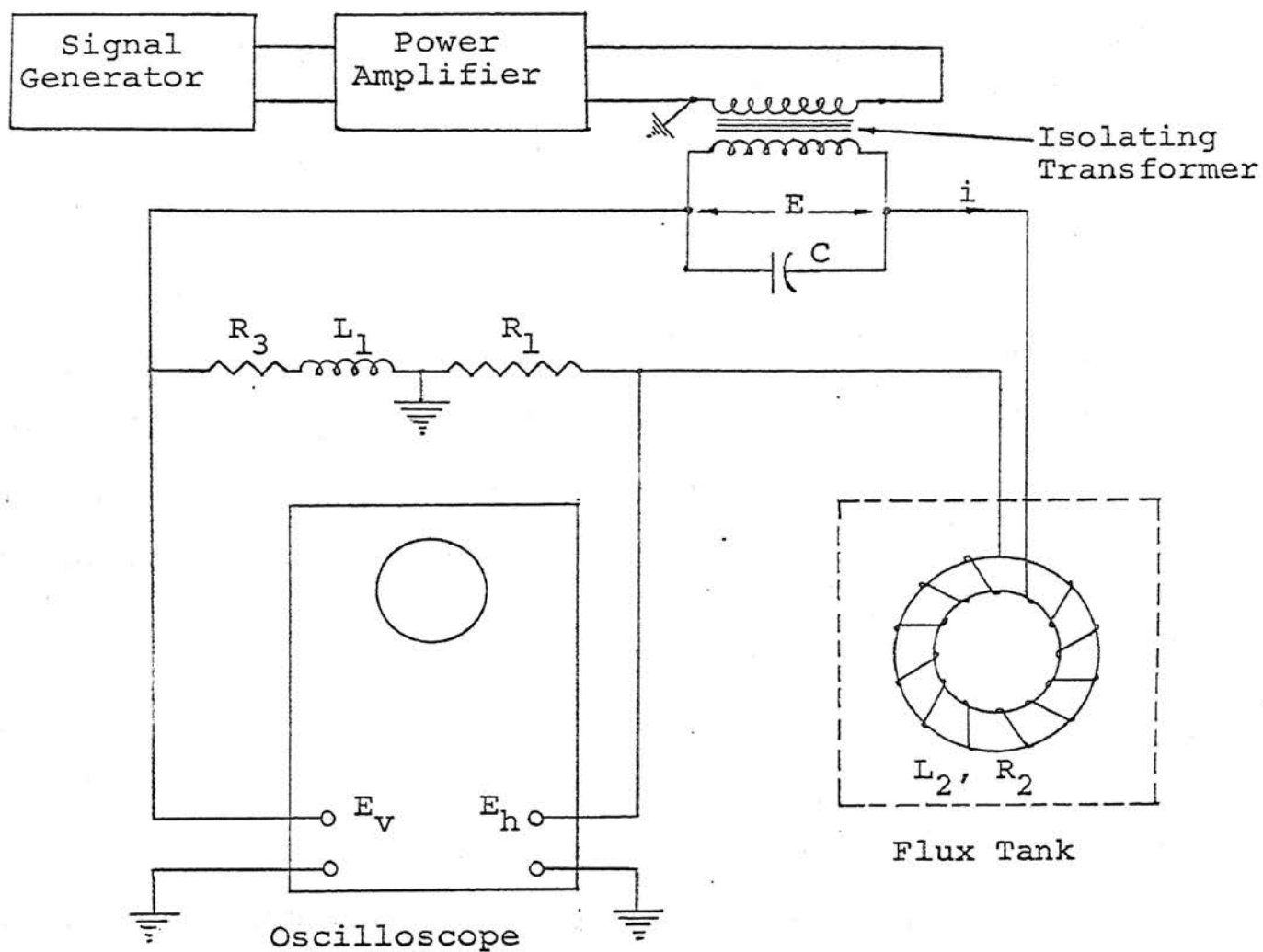
where

$$C_1 = \frac{0.4\pi N_b^2 A}{L_m} \times 10^{-8}$$

N_b = Number of turns of bias winding

A = Cross sectional area of core material

L_m = Mean length of the core



- R_1 and R_3 - Known Resistances
 R_2 - Winding Resistance
 L_1 - Known Air-Core Inductor
 L_2 - Winding Inductance
 C - High Frequency Filter Capacitor

Figure 3. Experimental Circuit for Measurement of Differential Permeability

After making the connections as shown in Figure 3 and adjusting the supply voltage as explained earlier, the oscilloscope photographs were taken (Figure 4a,b,c).

A sketch of such pattern is shown in Figure 4d which explains how the corresponding values of E_v and E_h were obtained from the oscilloscope pattern. Each pair of values of E_v and E_h yields one value of dB/dH , using equation 3.1. The value of H corresponding to each value of dB/dH was calculated from the corresponding value of E_h using the following equation,

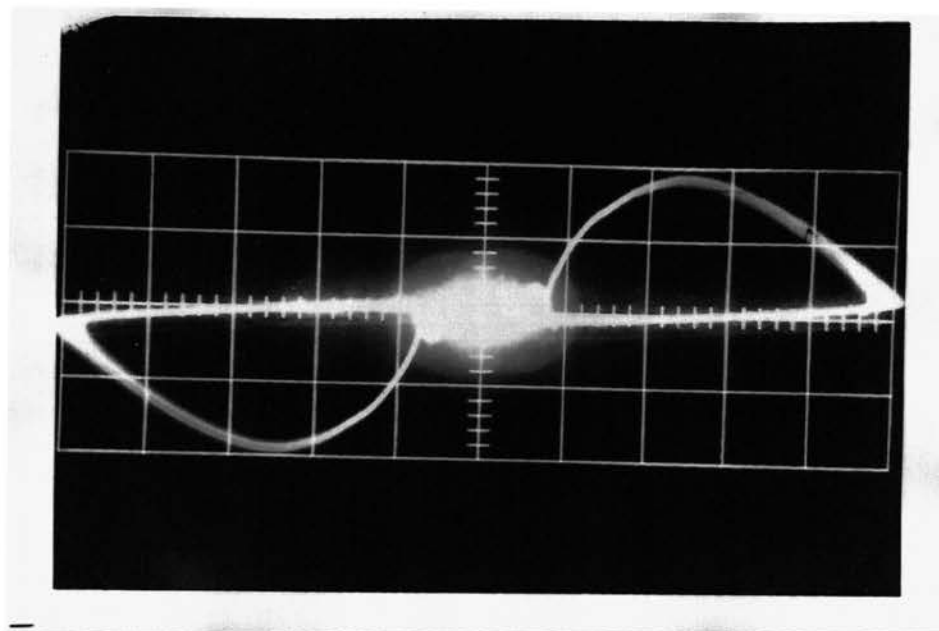
$$H = \frac{0.4\pi N_b (E_h/R_l)}{L_m} \quad (3.2)$$

Two bright dots near the origin (Points A and C in Figure 4d) on the E_h axis correspond to the coercive force of the core material. The coercive force of the material used here was found to be about 0.1 oersted. These dots are not clearly visible on the photographs as they were too bright. The portion of the oscilloscope pattern between points A and B and also between points C and D corresponds to the core in saturation.

A digital computer program was written to calculate dB/dH and H . The important data are shown in Table III. For the computer program see Appendix E.

The data of Table III were plotted on a log-log graph paper as shown in Figure 5.

An a.c. bridge was also tried to measure the differential permeability by measuring the inductance at various levels



Core - S-1000 E47-HA-1935-C

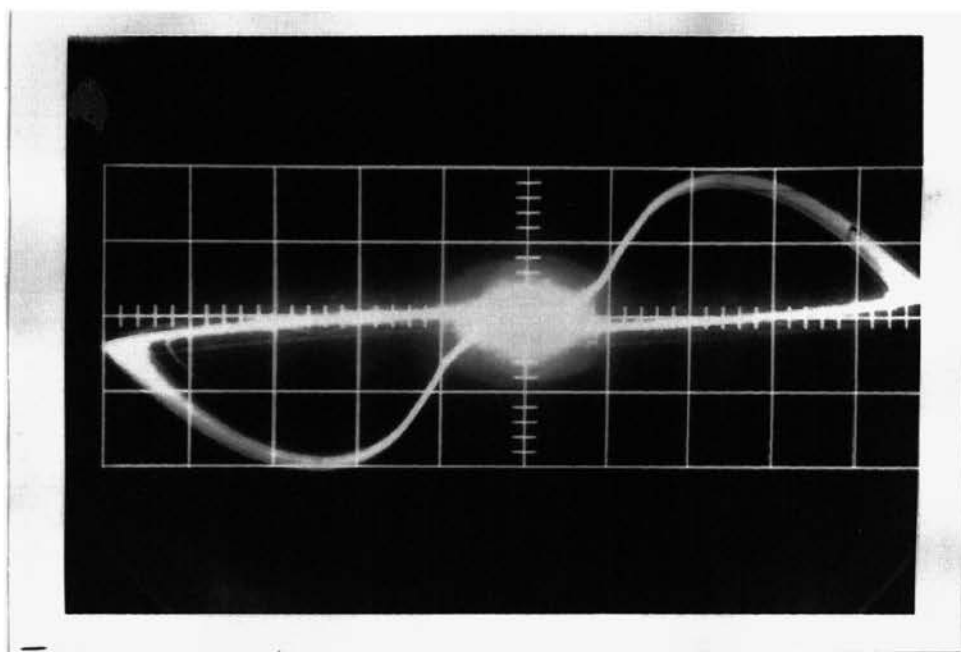
$R_1 = 47.5$ ohms, $R_2 = 5.65$ ohms, $R_3 = 3.1$ ohms

$L_1 = 274$ Micro-H, $E = 1.85$ volts

Horizontal Scale - 1 Volt = 3.4 cms.

Vertical Scale - 1 Volt = 3.3 cms.

Figure 4(a). Oscilloscope Waveform for Core S-1000 E47-HA-1935-C



Core - S-1000 E47-HB-1936-C

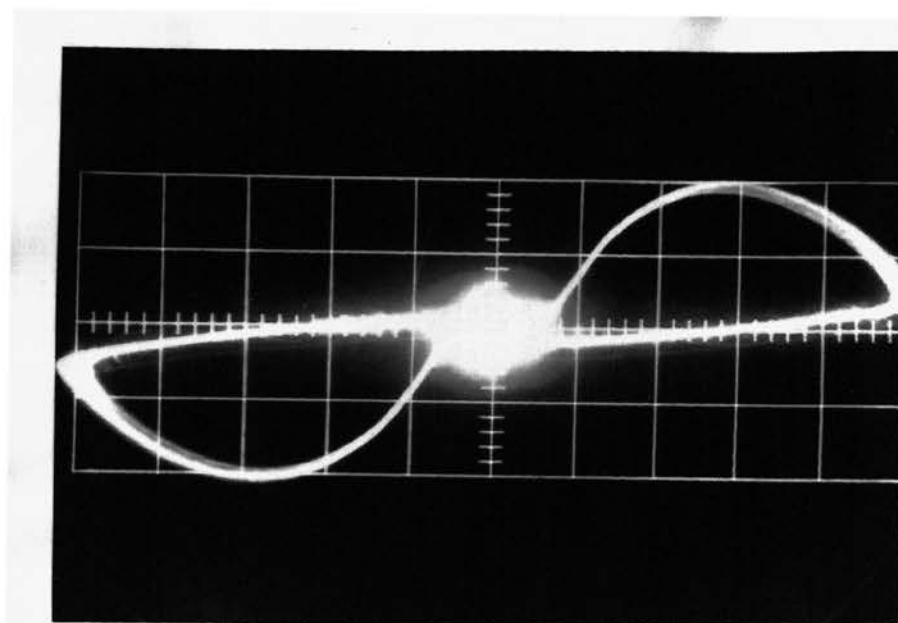
$R_1 = 48$ ohms, $R_2 = 5.62$ ohms, $R_3 = 3.0$ ohms

$L_1 = 274$ Micro-H, $E = 1.53$ volts

Horizontal Scale - 2 volts = 7.6 cms.

Vertical Scale - 0.5 volts = 2.5 cms.

Figure 4(b). Oscilloscope Waveform for Core S-1000 E47-HB-1936-C



Core - S-1000 E47-HC-1937-C

$R_1 = 47.5$ ohms, $R_2 = 5.5$ ohms, $R_3 = 3.1$ ohms

$L_1 = 274$ Micro-H, $E = 1.9$ volts

Horizontal Scale - 2 volts = 6.65 cms.

Vertical Scale - 0.5 volt = 2.85 cms.

Figure 4(c). Oscilloscope Waveform for Core S-1000 E47-HC-1937-C

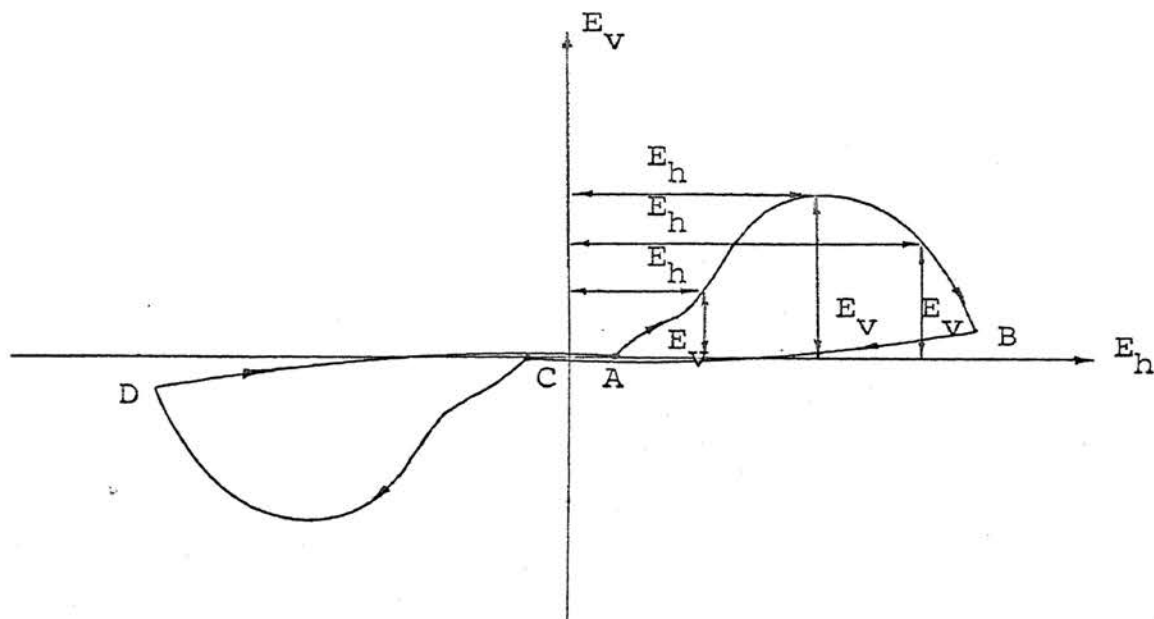


Figure 4(d). Typical Oscilloscope Pattern

of saturation and then using the equation D-4 to calculate dB/dH . However, this method was not quite so successful because of the following:

1. For incremental and differential permeabilities to be approximately equal, the oscillator level of the bridge was kept low. But at the low levels the bridge was not sensitive enough to measure the inductance.
2. When the oscillator level was increased so as to make the bridge sensitive enough, it was not possible to balance the bridge. This might be because at saturation the inductance and the quality factor of the core and winding were both low.

Table III. Differential Permeability for Various Degrees of Excitation

Sr. No.	S-1000 E47-HA-1935-C		S-1000 E47-HB-1936-C		S-1000 E47-HC-1937-C	
	H Oersteds	Differential Permeability	H Oersteds	Differential Permeability	H Oersteds	Differential Permeability
1	0.370	1212.8	0.255	5916.3	0.345	4548.3
2	0.411	931.8	0.364	1874.5	0.421	2598.7
3	0.494	622.7	0.437	1106.7	0.488	1818.7
4	0.617	439.7	0.547	668.5	0.589	1279.3
5	0.700	348.3	0.619	543.0	0.724	924.5
6	0.823	258.5	0.729	413.5	0.888	672.0
7	0.905	224.6	0.911	293.5	1.052	535.0
8	1.029	188.5	1.093	251.3	1.279	426.6
9	1.152	166.2	1.275	217.5	1.473	393.3
10	1.234	162.3	-	-	1.607	378.9

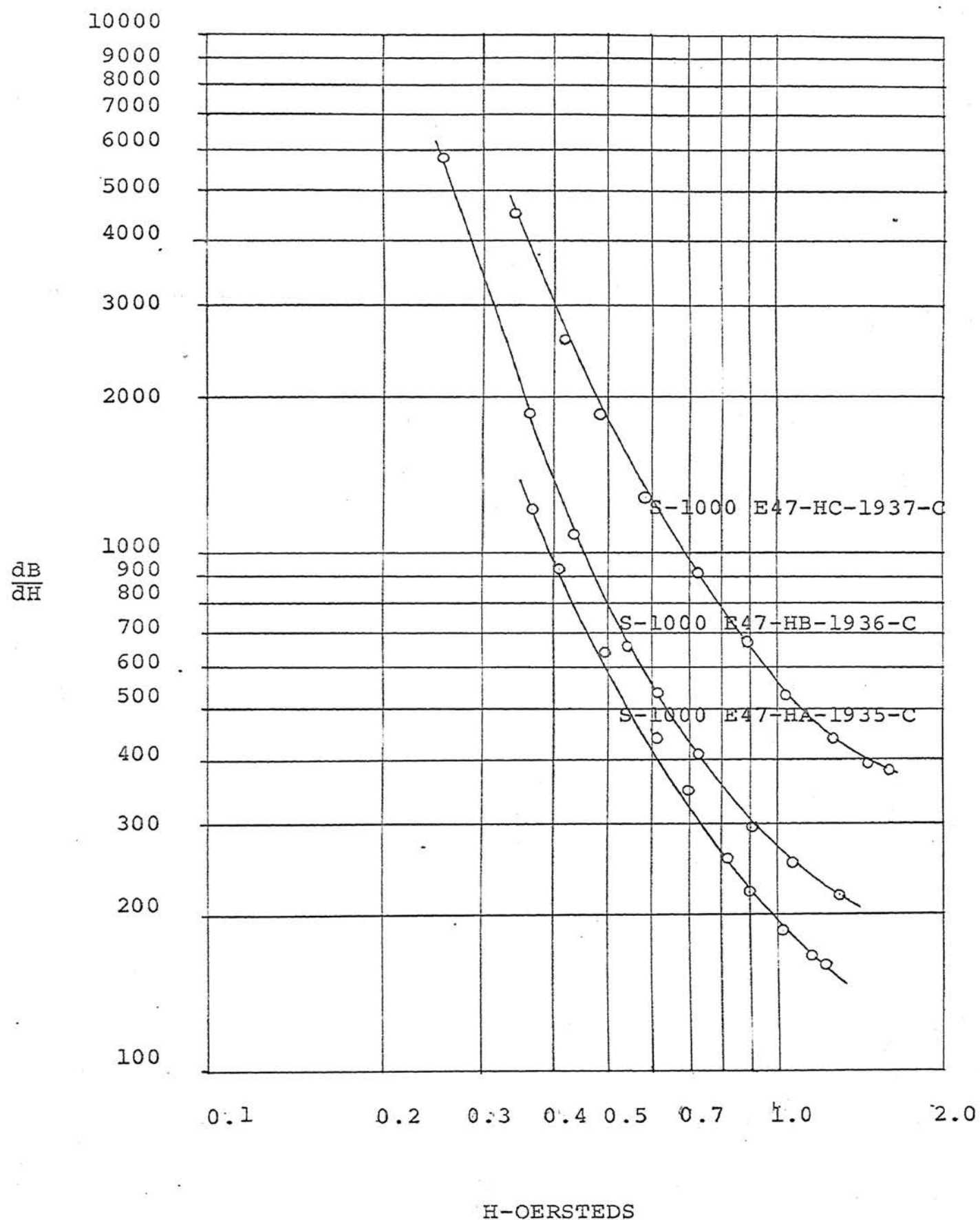


Fig. 5 DIFFERENTIAL PERMEABILITY VERSUS H

3. The inductance measured at best possible balance of the bridge was varying considerably with the oscillator level when the level was kept high.

For the above reasons the data taken from this a.c. bridge method were considered to be unreliable and not presented in this thesis.

IV. DETERMINATION OF DEMAGNETIZING FACTOR

Using Table II and Figure 5, the differential permeabilities for various degrees of saturation were obtained and Table IV was prepared. The data of Table IV were plotted as shown in Figure 6.

In view of equation 1.3 the data of the table IV, when plotted as in Figure 6, should lie on a straight line of unity slope. Also the intercept of this line on the vertical axis should be the demagnetizing factor.

However, the above theoretical inference was not achieved completely. As shown in Figure 6, the points lay almost on a straight line but the slope varied from 1.26 to 2.46 depending upon the type of core.

For more precise determination of the demagnetizing factor, the reasons for the slope being more than unity, were analyzed as follows.

As utmost care was taken in the measurement during the experiment, the errors in measurement were considered to be small compared to other errors.

One possible error considered was an assumption made in the derivation of the equation 2.1. As mentioned in Chapter 2, the apparent permeability was measured by applying an a.c. field superimposed on a d.c. field. A minor hysteresis loop was formed at different points on the magnetizing curve. It was assumed in Chapter 2 that the average

Table IV. $1/\mu_{a\Delta}$ and $1/\mu\Delta$ for various degrees of excitation

Sr. No.	S-1000 E47-HA-1935-C			S-1000 E47-HB-1936-C			S-1000 E47-HC-1937-C		
	H oersteds	$1/\mu_{a\Delta}$	$1/\mu\Delta$	H oersteds	$1/\mu_{a\Delta}$	$1/\mu\Delta$	H oersteds	$1/\mu_{a\Delta}$	$1/\mu\Delta$
1	0.363	0.00567	0.00077	0.363	0.00433	0.000533	0.484	0.003515	0.000526
2	0.484	0.00733	0.00155	0.484	0.00513	0.00116	0.726	0.004187	0.001081
3	0.605	0.00954	0.00238	0.605	0.00619	0.00181	0.968	0.00489	0.001667
4	0.726	0.01171	0.00322	0.726	0.00735	0.00247	1.21	0.00565	0.002222
5	0.847	0.0141	0.00408	0.847	0.00861	0.00322	1.451	0.006314	0.002532

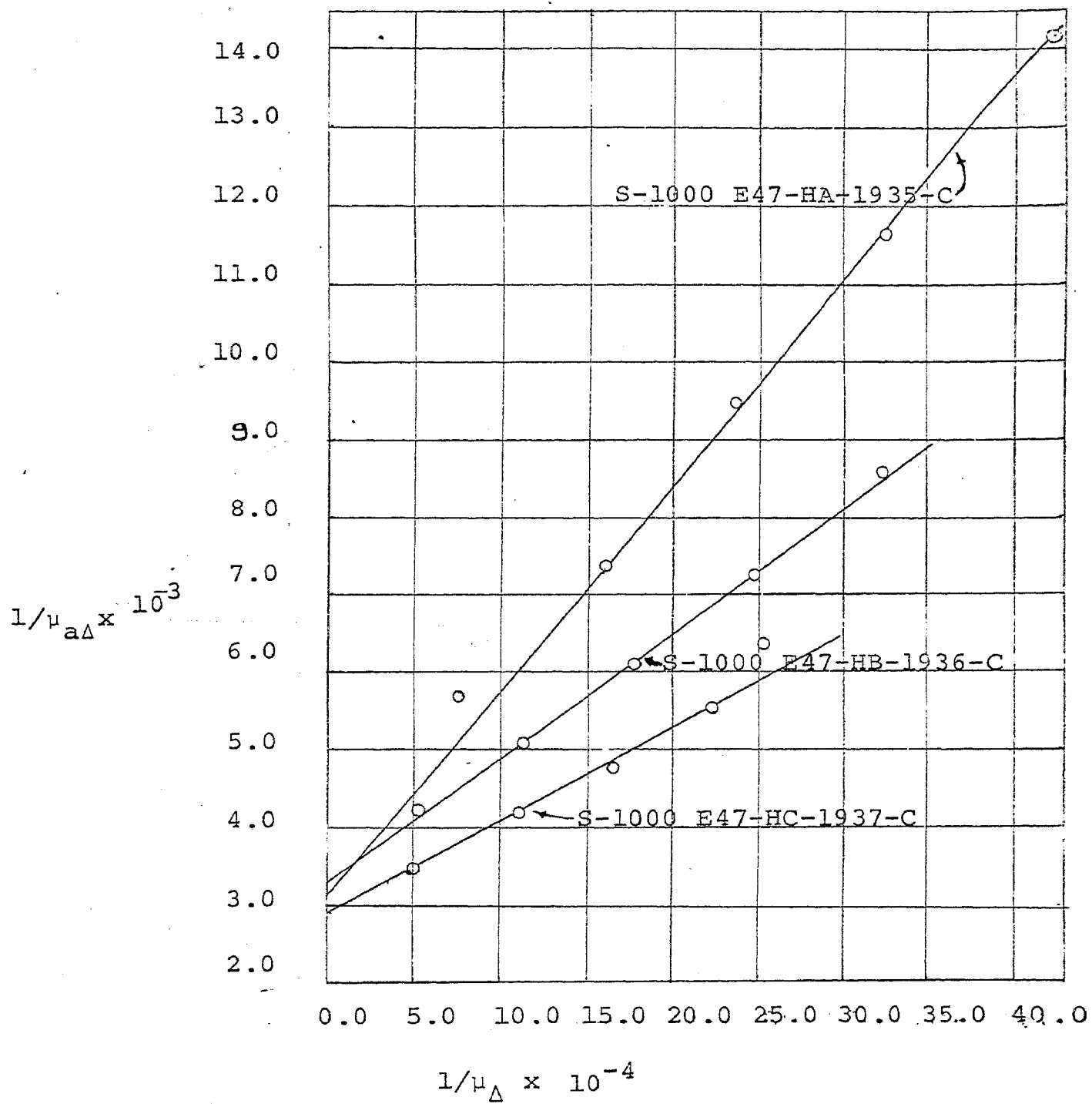


Fig. 6 $1/\mu_{a\Delta}$ VERSUS $1/\mu_{\Delta}$

slope of this minor loop was equal to the slope of the sheared magnetizing curve at that point.

Considering this as a major source of error, the straight lines of Figure 6 were redrawn (Figure 7), after reducing the ordinates by dividing them by their individual slopes. Figure 7 gave a second set of values of K.

One more set of values of K was obtained from an approximate relation derived from equation 1.3

$$1/\mu_{a\Delta} = 1/\mu_{\Delta} + K \quad (1.3)$$

But from Figure 5 it was judged that at low bias the differential permeability was very high and it was assumed that

$$1/\mu_{a\Delta} = K \quad \text{for zero bias} \quad (4.1)$$

Using the data of Table II along with the equation 4.1, the third set of values of K was obtained. All the three sets of values of K were tabulated in Table V.

From the Table V it was observed that the values of K given by sets 2 and 3 were quite close to each other except for the core s-1000 E47 - HA-1935-C. One possible reason for this was thought to be the very small tape thickness of that core (see Table I.) The ultrathin tape acts more like magnetic film and an error caused by not considering the film action in the various derivations might be considerable.

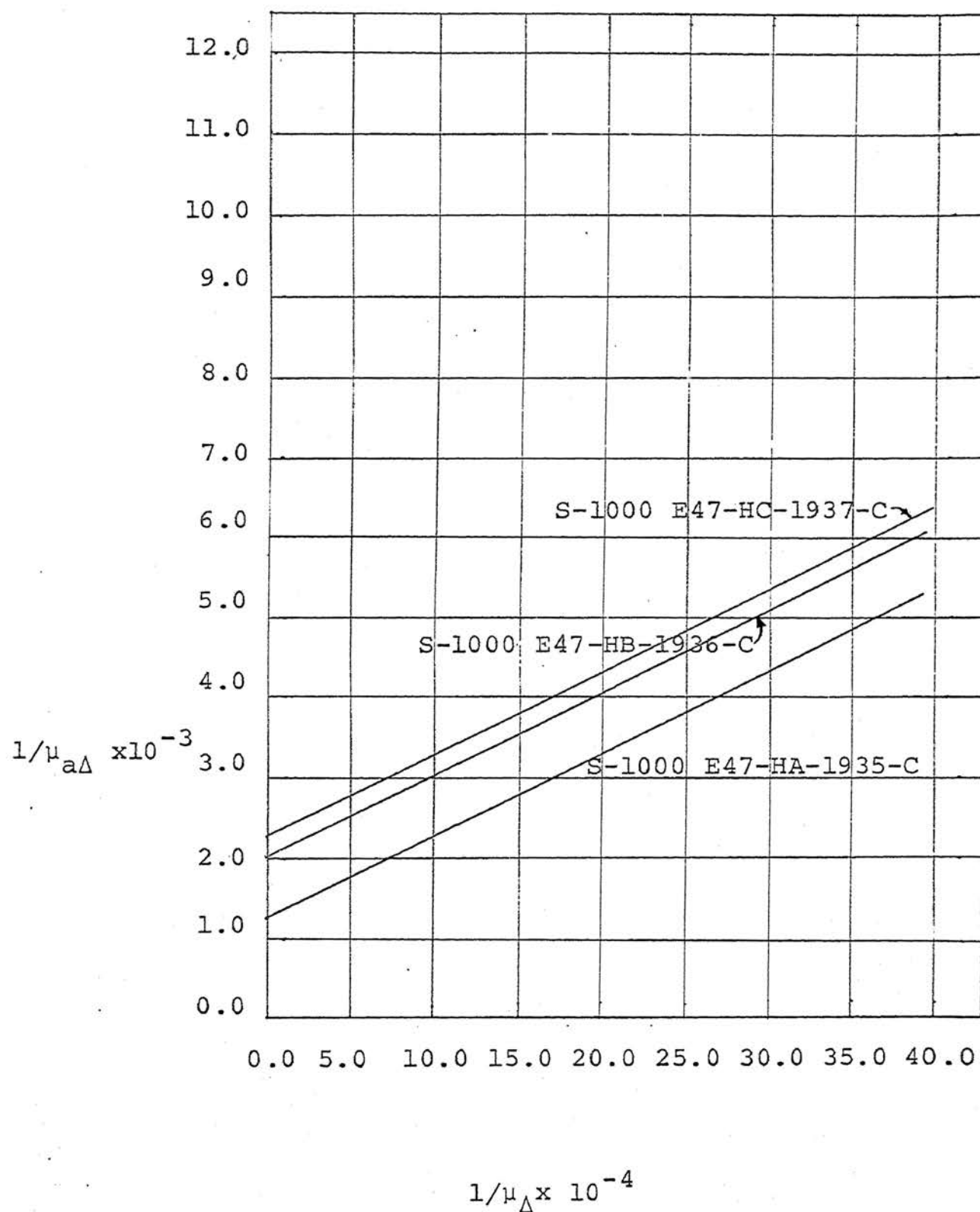


Figure 7. $1/\mu_{a\Delta}$ versus $1/\mu_{\Delta}$ after correction for unity slope.

Table V. Values of Demagnetizing factor 'K'

Sr. No.	Reference	S-100 E47-HA-1935-C		S-1000 E47-HB-1936-C		S-100 E47-HC-1937-C	
		K	Slope	K	Slope	K	Slope
1	Fig. 6	0.0032	2.46	0.00335	1.62	0.0029	1.126
2	Fig. 7	0.0013	1.00	0.00207	1.00	0.0023	1.00
3	Eq. 4.1	0.00285	-	0.00221	-	0.00222	-

Thus eventhough it was not possible to find the value of K for the core s-1000 E47-HA-1935-C within a reasonable accuracy, the values for the other cores lie in the following ranges,

Core s-1000 E47-HB-1936-C: $K=0.0021-0.0022$

Core s-100 E47-HC-1937-C: $K=0.0022-0.0023$

In 1967 Marshall¹³ gave an expression for the demagnetizing factor K as follows,

$$K = 3(\sqrt{A}/L_m)^{1.6} \quad (4.2)$$

The above expression was shown to be quite accurate for K much larger than those used here. However, for smaller K this expression should be modified. In view of the values of K obtained here the following expression was found accurate for the range of K used in this thesis,

$$K = 3(\sqrt{A}/L_m)^{1.51}$$

V. CONCLUSION AND RECOMMENDATIONS

For the ring-cores this thesis gives a method of measuring the demagnetizing factors. A similar but simplified method was used by Marshall on cores much bigger and having larger demagnetizing factors. Certain refinements were made in order to make the measurement on much smaller cores.

Possibly because of the magnetic film action of thin tape, the method appears to be unsuitable for cores wound with ultrathin tapes. The thinnest tape which gave quite accurate results with this method was 1/4 mil. thick.

As mentioned in Chapter 2, the core was not kept in the flux-tank during the measurement of apparent permeability. If the complete Helmholtz Coils with core were placed in the flux tank during this experiment, this should give better results as the effect of the earth's field would be eliminated almost completely.

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APPENDIX - A

DERIVATION OF EQUATION 1-3

Considering a cylindrical rod or a toroidal core of magnetic material, placed in an uniform magnetic field (Fig. 1) the attenuated signal H can be written as.

$$H = H_x - H_d$$

$$H + \Delta H = (H_x + \Delta H_x) - (H_d + \Delta H_d)$$

$$\Delta H = \Delta H_x - \Delta H_d$$

An increase in external field will bring about an increase in H within the material and an increase in H_d because of the increased polarization. The demagnetizing field is³,

$$H_d = K(B - H)$$

$$\Delta H_d = K \left(\Delta H \cdot \frac{\Delta B}{\Delta H} - \Delta H \right)$$

$$\Delta H = \Delta H_x - \Delta H_d$$

$$= \Delta H_x - K (\Delta H \mu_\Delta - \Delta H)$$

$$\frac{\Delta H}{\Delta B} = \frac{\Delta H_x}{\Delta B} - K \left(\mu_\Delta \frac{\Delta H}{\Delta B} - \frac{\Delta H}{\Delta B} \right)$$

$$1/\mu_\Delta = 1/\mu_{a\Delta} - K (1 - 1/\mu_\Delta)$$

Assuming $1/\mu_\Delta \ll 1$ the above expression reduces to:

$$1/\mu_{a\Delta} = 1/\mu_\Delta + K \quad (1-3)$$

APPENDIX - B

DERIVATION OF EQUATION 2.1

Considering Fig. 2, it can be written as follows,

$$\begin{aligned}
 e_0 &= 2 N_0 A (dB/dt) \cdot 10^{-8} \\
 &= 2 N_0 A (dB/dH_x) (dH_x/dt) \cdot 10^{-8} \\
 &= 2 N_0 A \mu_{a\Delta} (dH_x/dt) \cdot 10^{-8} \quad (B-1)
 \end{aligned}$$

But using the coil constant for the Helmholtz coils,

$$\begin{aligned}
 dH_x/dt &= 6.65 di_s/dt \\
 &= 6.65 (1/L) E_s \sin \omega t \\
 \therefore e_s &= E_s \sin \omega t = L di_s/dt \\
 &= \frac{6.65}{2.45} E_s \sin \omega t \\
 &= 2.77 E_s \sin \omega t \quad (B-2)
 \end{aligned}$$

Substituting equation (B-2) into equation (B-1) and solving for $\mu_{a\Delta}$, we get,

$$\begin{aligned}
 \mu_{a\Delta} &= \frac{10^8}{5.45 N_0 A} \times \frac{E_{op} \sin \omega t}{E_s \sin \omega t} \\
 &= \frac{1.835 \times 10^7}{N_0 A} \times \frac{E_0}{E_{is}} \quad (2.1)
 \end{aligned}$$

APPENDIX - C

PROGRAM FOR DETERMINATION OF APPARENT
PERMEABILITY

$$H = \frac{0.4\pi N_b B_i}{L_m}$$

$$\mu_{a\Delta} = \frac{1.835 E_o 10^7}{N_o A E_{is}} \quad (2.1)$$

The above equations were programmed for IBM 360 digital computer using Fortran IV language. The program is as follows,

```

/WAT4      EEEE490M,TIME=1,PAGES=10  D
C          MEASUREMENT OF APPARENT PERMEABILITY
1          DIMENSION EOM(20)
2          READ(1,4) EOI,A,BI,NB,NO,M
3          READ(1,1) (EOM(I),I=1,M)
4          X=1
5          WRITE(3,5)
6          20 WRITE(3,8) EOI,NB,NO
7          WRITE(3,2)
8          DO 10 I=1,M
9          H=(0.4*3.1416*NB*BI)/(8.0*1.0E3)
10         EO=SQRT(EOM(I)*EOM(I)-EOI*EOI)
11         U=(1.835*EO*1.0E7)/(NO*A*100)
12         Y=1/U
13         WRITE(3,3)H, EOM(I),U ,Y
14         10 BI=BI+2.0
15         IF(X-2)30,40,50
16         30 X=2
17         READ(1,4) EOI,A,BI,NB,NO,M
18         READ(1,1) (EOM(I),I=1,M)
19         WRITE(3,6)
20         GO TO 20
21         40 X=3
22         READ(1,4) EOI,A,BI,NB,NO,M
23         READ(1,1) (EOM(I),I=1,M)
24         WRITE(3,7)
25         GO TO 20

```

```

26      50 STOP
27      1 FORMAT (14F5.2)
28      2 FORMAT (8X,'H-OERSTEDS',5X,'EOM-MV',8X,'APP.
      1 PERM.',5X,'1/APP. PERM. ')
29      3 FORMAT (/10X,F5.3,7X,F6.2,10X,F6.2,10X,F7.5)
30      4 FORMAT (3F10.3,3I10)
31      5 FORMAT (//20X,'CORE - S-1000 E47-HA-1935-C')
32      6 FORMAT (//20X,'CORE - S-1000 E47-HB-1936-C')
23      7 FORMAT (//20X,'CORE - S-1000 E47-HC-1937-C')
34      8 FORMAT (//20X,'EOI=',F4.2,' NB=',I3,' NO=',I3)
35      END

```

/DATA

CORE - S-100 E47-HA-1935-C
EOI=0.92 NB=385 NO=850

H-OERSTEDS	EOM-MV	APP. PERM.	1/APP. PERM
0.000	8.25	350.48	0.00285
0.121	6.55	277.23	0.00361
0.242	5.20	218.79	0.00457
0.363	4.23	176.50	0.00567
0.484	3.32	136.37	0.00733
0.605	2.62	104.87	0.00954
0.726	2.20	85.43	0.01171
0.847	1.90	71.07	0.01407
0.968	1.66	59.07	0.01693

CORE - S-1000 E47-HB-1936-C
EOI=1.00 NB=385 NO=850

H-OERSTEDS	EOM-MV	APP. PERM.	1/APP. PERM
0.000	8.85	451.98	0.00221
0.121	6.65	337.93	0.00296
0.242	5.55	280.60	0.00356
0.363	4.60	230.79	0.00433
0.484	3.92	194.82	0.00513
0.605	3.30	161.65	0.00619
0.726	2.83	136.08	0.00735
0.847	2.47	116.09	0.00861
0.968	2.20	100.72	0.00993

CORE - S-100 E47-HC-1937-C

EOI=0.85 NB=385 No=850

H-OERSTEDS	EOM-MV	APP. PERM.	1/APP. PERM.
0.000	10.40	450.23	0.00222
0.121	8.06	348.15	0.00287
0.242	6.60	284.30	0.00352
0.363	5.56	238.67	0.00419
0.484	4.78	204.32	0.00489
0.605	4.16	176.89	0.00565
0.726	3.74	158.20	0.00632
0.847	3.33	139.85	0.00715

APPENDIX - D

DERIVATION OF EQUATION 3.1

Consider the circuit of Figure 3 during the short time near the peak of the voltage across the secondary of the isolating transformer. During this time the voltage can be considered as constant and the current increases rapidly as the core has just become saturated causing a rapid decrease in L_2 . We can write as follows,

$$E_h = i \cdot R_1$$

$$\therefore i = E_h / R_1$$

$$E_v = iR_3 + L_1 \, di/dt$$

$$= \frac{E_h \cdot R_3}{R_1} + L_1 \frac{di}{dt}$$

$$\therefore \frac{di}{dt} = (E_v - E_h \frac{R_3}{R_1}) \cdot \frac{1}{L_1} \quad (D-1)$$

At saturation,

$$E = (L_1 + L_2) \frac{di}{dt} + i (R_1 + R_2 + R_3)$$

Substituting for di/dt from equation (D-1) and writing R for $(R_1 + R_2)$, the above expression can be reduced to,

$$L_2 = \frac{L_1 (E - E_v - E_h \frac{R}{R_1})}{E_v - E_h \frac{R_3}{R_1}} \quad (D-2)$$

But from the definition of inductance,

$$L_2 = N_b \frac{d\phi}{di} - 10^{-8}$$

$$L_2 = N_b A \frac{dB}{dH} \cdot \frac{dH}{di} \cdot 10^{-8} \quad (D-3)$$

But,

$$H = \frac{0.4 \pi N_b i}{L_m}$$

Substituting this in the equation (D-3)

$$L_2 = \frac{0.4 \pi N_b^2 A}{L_m} \cdot \frac{dB}{dH} \cdot 10^{-8} \quad (D-4)$$

Comparing the equations D-2 and D-4,

$$\frac{dB}{dH} = \frac{L_1 (E - E_v - E_h R/R_1)}{C_1 (E_v - E_h R_3/R_1)} \quad (3.1)$$

$$\text{where } C_1 = \frac{0.4 \pi N_b^2 A}{L_m} \times 10^{-8}$$

APPENDIX - E

PROGRAM FOR DETERMINATION OF
DIFFERENTIAL PERMEABILITY

$$L_2 = \frac{L_1 (E - E_v - E_h R/R_1)}{E_v - E_h R_3/R_1} \quad (D-2)$$

$$dB/dH = \frac{L_2 L_m 10^8}{0.4\pi N_b^2 A} \quad \text{From (D-3)}$$

$$H = \frac{0.4\pi N_b i}{L_m}$$

The above equations were programmed for IBM 360 digital computer using Fortran IV Language. The program is as follows,

```

/WAT4  EEEE490M,TIME=1,PAGES=10  D  PATEL S.C.  JOB 496
C      MEASUREMENT OF DIFFERENTIAL PERMEABILITY
1      REAL L1, L2
2      DIMENSION EV(20),EH(20)
3      READ(1,4)E,R1,R2,R3,L1,A,NB,M
4      READ(1,1)(EV(I),EH(I),I=1,M)
5      X=1
6      WRITE(3,5)
7      CV=(1000*4)/(3.3*3.66)
8      CH=(1*10)/(3.4*9.1)
9      C=(0.4*3.1416*NB)/(8.0*1.0E3)
10     20 WRITE(3.8)E,R1,R2,R3,L1
11     WRITE(3,2)
12     R=R1+R2
13     DO 10I=1,M
14     L2=(L1*(E-CV*EV(I)/1.0E3-CH*EH(I)*R/R1))/(CV*EV(I)
4-CH*EH(I)*R3*1.0E3/R1)

```

```

15      UT=(L2 *8.0*1.0E8)/(0.4*3.1416*NB*NB*A)
16      BI=(CH*EH(I)*1.0E3)R1
17      H=C*BI
18  10  WRITE (3,3)H,UT,EV(I),EH(I)
19      IF (X-2)30,40,50
20  30  X=2
21      READ(1,4)E,R1,R2,R3,L1,A,NB,M
22      READ(1,1)(EV(I),EH(I),I=1,M)
23      CV=(500*4)/(2.5*3.66)
24      CH=(2*10)/(7.6*9.1)
25      WRITE(3.6)
26      GO TO 20
27  40  X=3
28      READ(1,4)E,R1,R2,R3,L1,A,NB,M
29      READ(1,1)(EV(I),EH(I),I=1,M)
30      CV=(500*4)/(2.85*3.66)
31      CH=(2*10)/(6.65*9.1)
32      WRITE(3.7)
33      GO TO 20
34  50  STOP
35      1  FORMAT (14F5.2)
36      2  FORMAT(/8X,'H-OERSTEDS',8X,'DIFFERENTIAL',7X,'EV-
1CMS',5X,'EH-CMS'/26X,'PERMEABILITY')
37      3  FORMAT(/10X,F5.3,12X,F9.3,9X,F5.2,6X,F5.2)
38      4  FORMAT(6F5.2,2I5)
39      5  FORMAT(//20X,'CORE - S-1000E47-HA-1935-C')
40      6  FORMAT(//20X,'CORE - S-1000E47-HB-1936-C')
41      7  FORMAT(//20X,'CORE - S-1000E47-HC-1937-C')
42      8  FORMAT(/10X,'E=',F4.2,'VOLTS R1=',F5.2,' OHMS R2=',
2F4.2,' OHMS R3=',F4.2,' OHMS L1=',F6.2)
43      END

```

/DATA

CORE - S-1000 E47-HA-1935-C

E=1.85 VOLTS R1=47.50 OHMS R2=5.65 OHMS R3=3.10
OHMS L1=274.00

H-OERSTEDS	DIFFERENTIAL PERMEABILITY	EV-CMS	EH-CMS
0.370	1212.836	0.79	0.90
0.411	931.825	0.95	1.00
0.494	622.675	1.22	1.20
0.617	439.723	1.43	1.50
0.700	348.279	1.56	1.70
0.823	258.466	1.68	2.00
0.905	224.581	1.69	2.20
1.029	188.529	1.65	2.50
1.152	166.232	1.55	2.80
1.234	162.343	1.44	3.00

CORE - S-1000 E47-HB-1936-C

E=1.53 VOLTS R1=48.00 OHMS R2=5.62 OHMS R3=3.00 OHMS
L1=274.00

H-OERSTEDS	DIFFERENTIAL PERMEABILITY	EV-CMS	EH-CMS
0.255	5916.305	0.32	0.70
0.364	1874.463	0.79	1.00
0.437	1106.749	1.14	1.20
0.547	668.504	1.50	1.50
0.619	543.045	1.62	1.70
0.729	413.490	1.73	2.00
0.911	293.463	1.72	2.50
1.083	251.313	1.47	3.00
1.275	217.537	1.15	3.50

CORE - S-1000 E47-HC-1937-C

E=1.90 VOLTS R1=47.50 OHMS R2=5.50 OHMS R3=3.10 OHMS

L1=274.00

H-OERSTEDS	DIFFERENTIAL PERMEABILITY	EV-CMS	EH-CMS
0.345	4548.254	0.50	0.82
0.421	2598.712	0.77	1.00
0.488	1818.667	1.00	1.16
0.589	1279.339	1.26	1.40
0.724	924.544	1.50	1.72
0.888	672.048	1.70	2.11
1.052	535.006	1.76	2.50
1.279	426.606	1.67	3.04
1.473	393.317	1.44	3.50
1.607	378.867	1.25	3.82

OBJECT CODE= 2728 BYTES,ARRAY AREA= 160 BYTES,UNUSED=
47112 BYTES

APPENDIX - F

NOMENCLATURE

- (1) A = Cross-sectional area of core material, cm^2 .
- (2) B = Flux density within core, gauss.
- (3) B_i = Bias current, amps.
- (4) E = Amplitude of voltage across the secondary of isolating transformer when it is adjusted as explained in chapter 3.
- (5) E_h = Voltage on horizontal deflecting plate of oscilloscope.
- (6) E_{is} = RMS input signal voltage.
- (7) E_o = Corrected RMS output voltage.
- (8) E_{op} = Peak value of output voltage.
- (9) E_{oi} = RMS output voltage without core in the output coil
- (10) E_{om} = Measured RMS output voltage.
- (11) E_s = Peak value of signal voltage.
- (12) E_v = Voltage on vertical deflecting place of oscilloscope.
- (13) e_o = Instantaneous output voltage.
- (14) e_s = Instantaneous signal voltage.
- (15) H = Magnetic field intensity, oersteds.
- (16) H_d = Demagnetizing field intensity, oersteds.
- (17) H_x = Ambient magnetic field intensity, oersteds.
- (18) i = Instantaneous current (see Figure 3)
- (19) i_s = Instantaneous current in signal circuit, amps.
- (20) K = Demagnetizing factor.
- (21) L = Inductance of Helmholtz coils, henries.
- (22) L_1 = Inductance (see Figure 3)
- (23) L_2 = Inductance (see Figure 3)
- (24) L_m = Mean length of core, cms.
- (25) N_b = Number of turns on bias winding.

- (26) N_o = Number of turns of output coil.
- (27) R_1 = Resistance (see Figure 3) ohms.
- (28) R_2 = Resistance (see Figure 3) ohms.
- (29) R_3 = Resistance (see Figure 3) ohms.
- (30) μ = True Permeability.
- (31) μ_a = Apparent permeability.
- (32) μ_Δ = Incremental true permeability.
- (33) $\mu_{a\Delta}$ = Incremental apparent permeability.

VITA

Subhashchandra Chimanlal Patel was born on October 7, 1942 in Sojitra, INDIA. He received his high school education from the New Era High School, Baroda, INDIA. After that he attended the Maharaja Sayajirao University of Baroda, INDIA, and obtained the degrees of Bachelor of Engineering (Electrical) and Bachelor of Engineering (Mechanical) in June, 1965 and June, 1966 respectively.

He was employed as an assistant electrical engineer with The Hindustan Brown Boveri Ltd., Baroda, INDIA, until August, 1967.

The author has been enrolled in the Graduate School of the University of Missouri at Rolla since September 1967 and has been a candidate for the Master of Science degree in Electrical Engineering.

He is a student member of the Institute of Electrical and Electronics Engineers.

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